LESSON 3.3b

Completing the Square with $ax^2 + bx + c, a \neq 1$

Today you will:

- Solve quadratic equations $ax^2 + bx + c = 0$ $(a \neq 1)$ by *Completing the Square*
- Put a quadratic into *vertex form* using *Completing the Square*
- Practice translating English to Math and vice-versa!

Core Vocabulary:

- Perfect square trinomial
- Completing the square, p. 112

What would be your first step in factoring $2x^2 + 8x + 8$?

- Factor out the GCF
- $2(x^2 + 4x + 4)$

What would be your first step in solving $2x^2 = 8$?

- Divide by the GCF
- $\frac{2x^2}{2} = \frac{8}{2}$

Solve $3x^2 + 12x + 15 = 0$ by completing the square.

SOLUTION

The coefficient *a* is not 1, so you must first divide each side of the equation by *a*.

$3x^2 + 12x + 15 = 0$	Write the equation.
$x^2 + 4x + 5 = 0$	Divide each side by 3.
$x^2 + 4x = -5$	Write left side in the form $x^2 + bx$.
$x^2 + 4x + 4 = -5 + 4$	Add $\left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = 4$ to each side.
$(x + 2)^2 = -1$	Write left side as a binomial squared.
$x + 2 = \pm\sqrt{-1}$	Take square root of each side.
$x = -2 \pm\sqrt{-1}$	Subtract 2 from each side.
$x = -2 \pm i$	Write in terms of *i*.

The solutions are $x = -2 + i$ and $x = -2 - i$.

Do problems Pg 116 #31-35 (part of your homework)

In Exercises 25–36, solve the equation by completing the square. *(See Examples 3 and 4.)*

31. $7t^2 + 28t + 56 = 0$

33. $5x(x + 6) = -50$

35. $4x^2 - 30x = 12 + 10x$

What is *vertex form* again?

- $y = a(x - h)^2 + k$

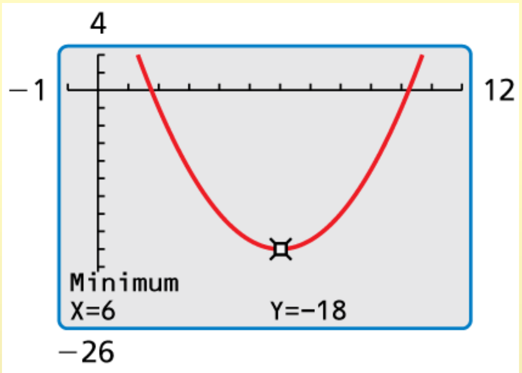
What do you have to do to convert *vertex form* into *standard form*?

- FOIL $(x - h)^2$ out and combine like terms
- $y = a[(x - h)(x - h)] + k$

Write $y = x^2 - 12x + 18$ in vertex form. Then identify the vertex.

SOLUTION

Check



$y = x^2 - 12x + 18$	Write the function.
$y + ? = (x^2 - 12x + ?) + 18$	Prepare to complete the square.
$y + 36 = (x^2 - 12x + 36) + 18$	Add $\left(\frac{b}{2}\right)^2 = \left(\frac{-12}{2}\right)^2 = 36$ to each side.
$y + 36 = (x - 6)^2 + 18$	Write $x^2 - 12x + 36$ as a binomial squared.
$y = (x - 6)^2 - 18$	Solve for y.

The vertex form of the function is $y = (x - 6)^2 - 18$. The vertex is $(6, -18)$.

Do problems Pg 116 #55-59 odd (part of your homework … yes I know these are out of order)

In Exercises 55–62, write the quadratic function in vertex form. Then identify the vertex. *(See Example 5.)*

55. $f(x) = x^2 - 8x + 19$

56. $g(x) = x^2 - 4x - 1$

57. $g(x) = x^2 + 12x + 37$

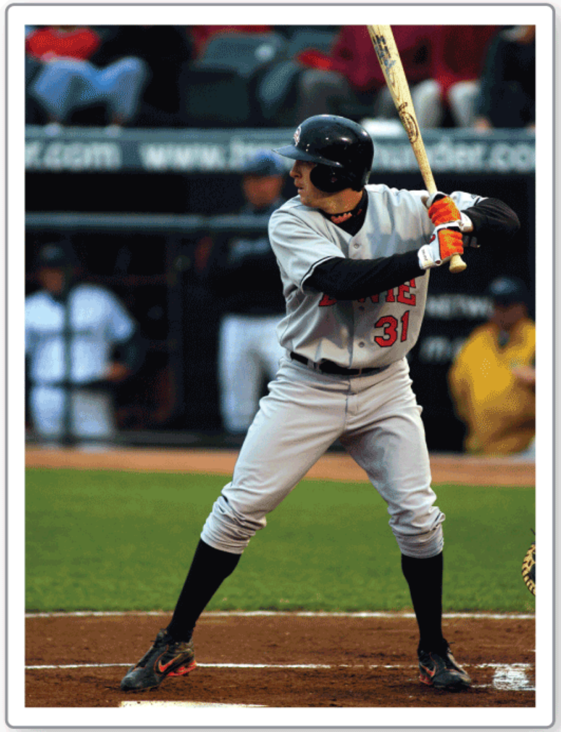
58. $h(x) = x^2 + 20x + 90$

59. $h(x) = x^2 + 2x - 48$

The height y (in feet) of a baseball t seconds after it is hit can be modeled by the function

$$y = -16t^2 + 96t + 3.$$

Find the maximum height of the baseball.
How long does the ball take to hit the ground?



SOLUTION

1. **Understand the Problem** You are given a quadratic function that represents the height of a ball. You are asked to determine the maximum height of the ball and how long it is in the air.

2. **Make a Plan** Write the function in vertex form to identify the maximum height. Then find and interpret the zeros to determine how long the ball takes to hit the ground.

3. Solve the Problem Write the function in vertex form by completing the square.

$y = -16t^2 + 96t + 3$	Write the function.
$y = -16(t^2 - 6t) + 3$	Factor -16 from first two terms.
$y + ? = -16(t^2 - 6t + ?) + 3$	Prepare to complete the square.
$y + (-16)(9) = -16(t^2 - 6t + 9) + 3$	Add $(-16)(9)$ to each side.
$y - 144 = -16(t - 3)^2 + 3$	Write $t^2 - 6t + 9$ as a binomial squared.
$y = -16(t - 3)^2 + 147$	Solve for y.

ANOTHER WAY

You can use the coefficients of the original function $y = f(x)$ to find the maximum height.

$$f\left(-\frac{b}{2a}\right) = f\left(-\frac{96}{2(-16)}\right) = f(3) = 147$$

The vertex is (3, 147).

$0 = -16(t - 3)^2 + 147$	Substitute 0 for y.
$-147 = -16(t - 3)^2$	Subtract 147 from each side.
$9.1875 = (t - 3)^2$	Divide each side by -16.
$\pm\sqrt{9.1875} = t - 3$	Take square root of each side.
$3 \pm \sqrt{9.1875} = t$	Add 3 to each side.

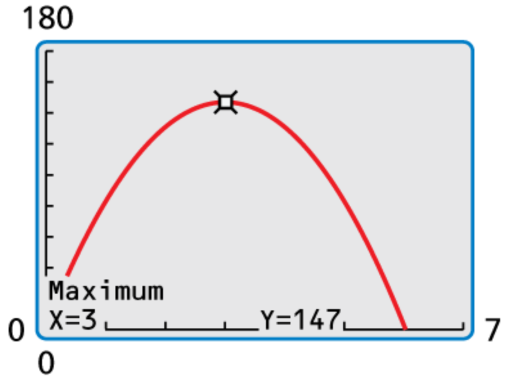
Reject the negative solution, $3 - \sqrt{9.1875} \approx -0.03$, because time must be positive.

So, the maximum height of the ball is 147 feet, and it takes $3 + \sqrt{9.1875} \approx 6$ seconds for the ball to hit the ground.

LOOKING FOR STRUCTURE

You could write the zeros as $3 \pm \frac{7\sqrt{3}}{4}$, but it is easier to recognize that $3 - \sqrt{9.1875}$ is negative because $\sqrt{9.1875}$ is greater than 3.

4. Look Back The vertex indicates that the maximum height of 147 feet occurs when $t = 3$. This makes sense because the graph of the function is parabolic with zeros near $t = 0$ and $t = 6$. You can use a graph to check the maximum height.



Homework

Pg 116 #31-63 odd, 64